CBCS SCHEME

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17CS36

Third Semester B.E. Degree Examination, June/July 2023
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Define the following with an example for each :
(i) Proposition.
(ii) Tautology.
(iii) Contradiction. **(06 Marks)**

b. Establish the validity of the argument :

$p \to q$
$q \to (r \wedge s)$
$\neg r \to (\neg t \vee u)$
$p \wedge t$
―――――
$\therefore u$ **(07 Marks)**

c. Let $p(x) : x^2 - 7x + 10 = 0$; $q(x) : x^2 - 2x - 3 = 0$; $r(x) : x < 0$. Find the truth value of the following statements, if universe contains only the integers 2 and 5.
(i) $\forall x, p(x) \to \neg r(x)$ (ii) $\forall x, q(x) \to r(x)$
(iii) $\exists x, p(x) \to r(x)$ (iv) $\exists x, q(x) \to r(x)$ **(07 Marks)**

OR

2 a. For any three propositions p, q, r, prove that $[p \to (q \wedge r)] \Leftrightarrow [(p \to q) \wedge (p \to r)]$ using truth table. **(06 Marks)**

b. Establish the validity of the following argument using rules of inference. If the band could not play rock music or the refreshments were not served on time, then the new year party could have been cancelled and Alica would have been angry. If the party were cancelled then refunds would have to be made. No refunds were made, therefore the band could play rock music. **(07 Marks)**

c. Give (i) direct proof (ii) indirect proof (iii) proof by contradiction for the statement "square of an odd integer, is an odd integer." **(07 Marks)**

Module-2

3 a. Prove by mathematical induction that, for every positive integer n, 5 divides $n^5 - n$. **(06 Marks)**

b. Find the number of permutations of the letters of the word MASSASAUGA. In how many of these all four A's are together? How many of them begin with S? **(07 Marks)**

c. In how many ways can one distribute eight identical balls into four distinct containers so that, (i) No container is left empty.
(ii) The fourth container gets an odd number of balls. **(07 Marks)**

OR

4 a. Let $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ and $a_n = a_{n-1}+a_{n-2}+a_{n-3}$ for $n \geq 3$. Prove that $a_n \leq 3^n$, for all positive integer n. **(06 Marks)**

b. A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations :
   (i) There is not restriction on the choice.
   (ii) Two particular persons will not attend separately.
   (iii) Two particular persons will not attend together. **(07 Marks)**

c. Determine the coefficients of,
   (i) $x^{11}y^4$ in the expansion of $(2x^3 - 3xy^2 + z^2)^6$.
   (ii) The constant term in the expansion of $\left(3x^2 - \frac{2}{x}\right)^{15}$
   (iii) $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(07 Marks)**

Module-3

5 a. Let $f : R \rightarrow R$ be defined by $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$. Determine (i) f(0) (ii) $f\left(\frac{5}{3}\right)$ (iii) $f^{-1}([-5,5])$. **(06 Marks)**

b. State the pigeonhole principle and generalization of pigeon hole principle. Prove that if 30 dictionaries in a library contains a total of 61,327 pages, then at least one of the dictionaries must have at least 2045 pages. **(07 Marks)**

c. Define partially ordered set. Draw the Hasse diagram representing the positive divisors of 36. **(07 Marks)**

OR

6 a. Let f, g, h be functions from R to R defined by $f(x) = x^2$, $g(x) = x+5$, $h(x) = \sqrt{x^2 + 2}$, verify that $(hog)of = ho(gof)$. **(06 Marks)**

b. Define equivalence relation. Let $A = \{1, 2, 3, 4, 5\}$. Define a relation R on $A \times A$ by (x_1, y_1) R $(x_2\ y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.
   (i) Verify that R is an equivalence relation on $A \times A$.
   (ii) Determine equivalence classes [(1, 3)], [(2, 4)] and [(1, 1)]. **(07 Marks)**

c. Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only if a is multiple of b, represent the relation R as matrix and draw its digraph. **(07 Marks)**

Module-4

7 a. Determine the number of positive intergers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. **(06 Marks)**

b. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have apple. The boy B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(07 Marks)**

c. If $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 37$ satisfies the recurrence relation $a_{n+2} + ba_{n+1} + ca_n = 0$ for $n \geq 0$, find the constants b and c, and also solve the relation a_n. **(07 Marks)**

OR

8 a. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(06 Marks)**

b. By using expansion formula, obtain the rook polynomial for the board in the Fig. Q8 (b). **(07 Marks)**

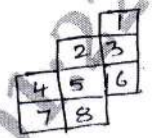

Fig. Q8 (b)

c. Solve the recurrence relation : $a_{n+1} = 3a_n + 5 \times 7^{n+1}$ for $n \geq 0$, given $a_0 = 2$. **(07 Marks)**

Module-5

9 a. Define isomorphism of two graphs, hence determine whether the graphs, shown in the Fig. Q9 (a) are isomorphic or not.

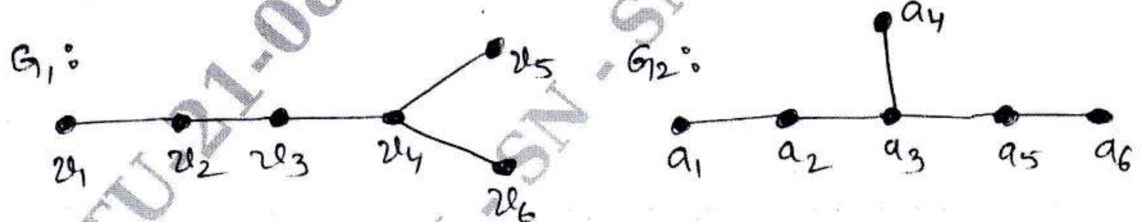

Fig. Q9 (a) **(06 Marks)**

b. Define complete graph and complete bipartite graph, and hence draw

(i) Kuratowaski's first graph K_5.

(ii) Kuratowaski's second graph $K_{3,3}$

(iii) 3-regular graph with 8 vertices.

(iv) Star graph $K_{1,5}$ **(07 Marks)**

c. Construct an optimal prefix code for the letters of the word ENGINEERING. Hence deduce the code for this word. **(07 Marks)**

OR

10 a. Define tree and prove that tree with n vertices has n-1 edges. **(06 Marks)**

b. Merge sort the list using trees,
-1, 7, 4, 11, 5, -8 ,15, -3, -2, 6, 10, 3. **(07 Marks)**

c. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occurs with frequencies 20, 28, 4, 17, 12, 7 respectively. **(07 Marks)**

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